

Non Singular Origin of the Universe and the Cosmological Constant Problem (CCP)*

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Abstract

We consider a non singular origin for the Universe starting from an Einstein static Universe in the framework of a theory which uses two volume elements $\sqrt{-g}d^4x$ and Φd^4x , where Φ is a metric independent density, also curvature, curvature square terms, first order formalism and for scale invariance a dilaton field ϕ are considered in the action. In the Einstein frame we also add a cosmological term that parametrizes the zero point fluctuations. The resulting effective potential for the dilaton contains two flat regions, for $\phi \rightarrow \infty$ relevant for the non singular origin of the Universe and $\phi \rightarrow -\infty$, describing our present Universe. Surprisingly, avoidance of singularities and stability as $\phi \rightarrow \infty$ imply a positive but small vacuum energy as $\phi \rightarrow -\infty$. Zero vacuum energy density for the present universe is the "threshold" for universe creation.

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The "Cosmological Constant Problem" [1], [2],[3] (CCP), is a consequence of the uncontrolled UV behavior of the zero point fluctuations in Quantum Field Theory (QFT), which leads to an equally uncontrolled vacuum energy density or cosmological constant term (CCT). This CCT is undetermined in QFT, but it is naturally very large, unless a delicate balance of huge quantities, for some unknown reason, conspires to give a very small final result. Here we will explore a candidate mechanism where the CCT is controlled, in the context of a very specific framework, by the requirement of a non singular origin for the universe.

We will adopt the very attractive "Emergent Universe" scenario, where conclusions concerning singularity theorems can be avoided [4], [5], [6], [7], [8], [9], [10], [11] by violating the geometrical assumptions of these theorems. In this scenario [4],[5] we start at very early times ($t \rightarrow -\infty$) with a closed static Universe (Einstein Universe).

In [4] even models based on standard General Relativity, ordinary matter and minimally coupled scalar fields were considered and can provide indeed a non singular (geodesically complete) inflationary universe, with a past eternal Einstein static Universe that eventually evolves into an inflationary Universe.

Those most simple models suffer however from instabilities, associated with the instability of the Einstein static universe. The instability is possible to cure by going away from GR, considering non perturbative corrections to the Einstein's field equations in the context of the loop quantum gravity[6], a brane world cosmology [7], considering the Starobinski model for radiative corrections (which cannot be derived from an effective action)[8] or exotic matter[9]. In addition to this, the consideration of a Jordan Brans Dicke model also can provide a stable initial state for the emerging universe scenario [10], [11].

In this essay we discuss a different theoretical framework, presented in details in ref.[12], where such emerging universe scenario is realized in a natural way, where instabilities are avoided and a succesfull inflationary phase with a gracefull exit can be achieved . The model we will use was studied first in [13] (in ref.[12] a few typos in [13] have been corrected and also the discussion of some notions discussed there as well has been improved), however, we differ with [13] in our choice of the state with (here and in ref.[12] with a lower vacuum energy density) that best represents the present state of the universe. This is crucial, since as it should be obvious, the discussion of the CCP depends crucially on what vacuum we take. We will express the stability and existence conditions for the non singular initial universe in

terms of the energy of the vacuum of our candidate for the present Universe.

We work in the context of a theory built along the lines of the two measures theory (TMT) [14], [15], [16], [17] which deals with actions of the form,

$$S = \int L_1 \sqrt{-g} d^4x + \int L_2 \Phi d^4x \quad (1)$$

where Φ is an alternative "measure of integration", a density independent of the metric, for example in terms of four scalars φ_a ($a = 1, 2, 3, 4$), it can be obtained as follows:

$$\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \varphi_a \partial_\nu \varphi_b \partial_\alpha \varphi_c \partial_\beta \varphi_d \quad (2)$$

and more specifically work in the context of the globally scale invariant realization of such theories [15], [16], which require the introduction of a dilaton field ϕ . We look at the generalization of these models [16] where an " R^2 term" is present,

$$L_1 = U(\phi) + \epsilon R(\Gamma, g)^2 \quad (3)$$

$$L_2 = \frac{-1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \quad (4)$$

$$R(\Gamma, g) = g^{\mu\nu} R_{\mu\nu}(\Gamma), R_{\mu\nu}(\Gamma) = R^\lambda_{\mu\nu\lambda} \quad (5)$$

$$R^\lambda_{\mu\nu\sigma}(\Gamma) = \Gamma^\lambda_{\mu\nu,\sigma} - \Gamma^\lambda_{\mu\sigma,\nu} + \Gamma^\lambda_{\alpha\sigma} \Gamma^\alpha_{\mu\nu} - \Gamma^\lambda_{\alpha\nu} \Gamma^\alpha_{\mu\sigma}. \quad (6)$$

global scale invariance is satisfied if [16], [15] (f_1, f_2, α being constants),

$$V(\phi) = f_1 e^{\alpha\phi}, U(\phi) = f_2 e^{2\alpha\phi} \quad (7)$$

In the variational principle $\Gamma^\lambda_{\mu\nu}, g_{\mu\nu}$, the measure fields scalars φ_a and the "matter" - scalar field ϕ are all to be treated as independent variables although the variational principle may result in equations that allow us to solve some of these variables in terms of others, that is, the first order formalism is employed, where any relation between the connection coefficients and the metric is obtained from the variational principle, not postulated a priori. A particularly interesting equation is the one that arises from the φ_a fields, this yields $L_2 = M$, where M is a constant that spontaneously breaks scale invariance. the Einstein frame, which is a redefinition of the metric by a conformal factor, is defined as

$$\bar{g}_{\mu\nu} = (\chi - 2\kappa\epsilon R) g_{\mu\nu} \quad (8)$$

where χ is the ratio between the two measures, $\chi = \frac{\Phi}{\sqrt{-g}}$, determined from the consistency of the equations to be $\chi = \frac{2U(\phi)}{M+V(\phi)}$. The relevant fact is that the connection coefficient equals the Christoffel symbol of this new metric (for the original metric this "Riemannian" relation does not hold). There is a "k-essence" type effective action, where one can use this Einstein frame metric. As it is standard in treatments of theories with non linear kinetic terms or k-essence models[18]-[21], it is determined by a pressure functional, ($X = \frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$).

$$S_{eff} = \int \sqrt{-\bar{g}}d^4x \left[-\frac{1}{\kappa}\bar{R}(\bar{g}) + p(\phi, R) \right] \quad (9)$$

$$p = \frac{\chi}{\chi - 2\kappa\epsilon R}X - V_{eff} \quad (10)$$

where V_{eff} is an effective potential for the dilaton field given by

$$V_{eff} = \frac{\epsilon R^2 + U}{(\chi - 2\kappa\epsilon R)^2} \quad (11)$$

\bar{R} is the Riemannian curvature scalar built out of the bar metric, R on the other hand is the non Riemannian curvature scalar defined in terms of the connection and the original metric, which turns out to be given by $R = \frac{-\kappa(V+M) + \frac{\kappa}{2}\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\chi}{1 + \kappa^2\epsilon\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}$. This R can be inserted in the action (9) or alternatively, R in the action (9) can be treated as an independent degree of freedom, then its variation gives the required value as one can check (which can then be reinserted in (9)). Introducing this R into the expression (11) and considering a constant field ϕ we find that V_{eff} has two flat regions. The existence of two flat regions for the potential is shown to be consequence of the s.s.b. of the scale symmetry (that is of considering $M \neq 0$). The quantization of the model can proceed from (9) (see discussion in [12]) and additional terms could be generated by radiative corrections. We will focus only on a possible cosmological term in the Einstein frame added (due to zero point fluctuations) to (9), which leads then to the new action

$$S_{eff,\Lambda} = \int \sqrt{-\bar{g}}d^4x \left[-\frac{1}{\kappa}\bar{R}(\bar{g}) + p(\phi, R) - \Lambda \right] \quad (12)$$

This addition to the effective action leaves the equations of motion of the scalar field unaffected, but the gravitational equations acquire a cosmological constant. Adding the Λ term can be regarded as a redefinition of $V_{eff}(\phi, R)$

$$V_{eff}(\phi, R) \rightarrow V_{eff}(\phi, R) + \Lambda \quad (13)$$

In this resulting model, there are two possible types of emerging universe solutions, for one of those, the initial Einstein Universe (realized in the region $\phi \rightarrow \infty$) can be stabilized due to the nonlinearities of the model, if $\epsilon < 0$, $f_2 > 0$ and $f_2 + \kappa^2 \epsilon f_1^2 > 0$ provided the vacuum energy density of the ground state, realized in the region $\phi \rightarrow -\infty$, being given by $V_{eff} \rightarrow \frac{1}{4\epsilon\kappa^2} + \Lambda = \Delta\lambda$ is positive, but not very large, since it should be bounded from above by the inequality $\Delta\lambda < \frac{1}{12(-\epsilon)\kappa^2} \left[\frac{f_2}{f_2 + \kappa^2 \epsilon f_1^2} \right]$. These are very satisfactory results, since it means that the existence and stability of the emerging universe prevents the vacuum energy in the present universe from being very large, but requires it to be positive. The transition from the emergent universe to the ground state goes through an intermediate inflationary phase, therefore reproducing the basic standard cosmological model as well. So, it turns out that the creation of the universe can be considered as a "threshold event" for zero present vacuum energy density, which naturally gives a positive but small vacuum energy density for the present universe.

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- [1] S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989).
 - [2] Y. Jack Ng, *Int. J. Mod. Phys.* **D1**, 145 (1992).
 - [3] S. Weinberg, astro-ph/0005265.
 - [4] G.F.R. Ellis and R. Maartens, *Class. Quantum Grav.* **21**, 223 (2004).
 - [5] G.F.R. Ellis, J. Murugan and C.G. Tsagas, *Class. Quantum Grav.* **21**, 233 (2004).
 - [6] D.J. Mulryne, R. Tavakol, J.E. Lidsey and G.F.R. Ellis, *Phys. Rev.* **D71**, 123512 (2005).
 - [7] A. Banerjee, T. Bandyopadhyay and S. Chakraborty, *Grav.Cosmol.***13**, 290,(2007).
 - [8] S. Mukherjee, B.C.Paul, S.D. Maharaj and A. Beesham, arXiv:qr-qc/0505103.
 - [9] S. Mukherjee, B.C.Paul, N.K. Dadhich, S.D. Maharaj and A. Beesham, *Class. Quantum Grav.* **23**, 6927 (2006).
 - [10] S. del Campo, R. Herrera and P. Labrana, *JCAP* **0907**, 006 (2009).
 - [11] S. del Campo, R. Herrera and P. Labrana, *JCAP* **0711**, 030 (2007).
 - [12] E.I. Guendelman, "Non Singular Origin of the Universe and its Present Vacuum Energy Density", e-Print: arXiv:1103.1427 [gr-qc], to appear in *Int. J. Mod. Phys. A*.
 - [13] S. del Campo, E.I. Guendelman, R. Herrera and P. Labrana, *JCAP* **1006**, 026 (2010).
 - [14] Basic idea is developed in E.I. Guendelman and A.B. Kaganovich, *Phys. Rev.* **D60**, 065004

- (1999).
- [15] For a recent review and further references see E.I. Guendelman, A.B. Kaganovich, Plenary talk given at the Workshop on Geometry, Topology, QFT and Cosmology, Paris, France, 28-30 May 2008. e-Print: arXiv:0811.0793 [gr-qc].
 - [16] E.I. Guendelman, *Mod. Phys. Lett.* **A14**, 1043 (1999), e-Print: gr-qc/9901017.
 - [17] E.I. Guendelman and O. Katz, *Class. Quantum Grav.* **20**, 1715 (2003), e-Print: gr-qc/0211095.
 - [18] T. Chiba, T.Okabe and M Yamaguchi, *Phys.Rev.D***62** 023511 (2000).
 - [19] C. Armendariz-Picon, V. Mukhanov and P.J. Steinhardt, *Phys.Rev.Lett.* **85** 4438, 2000.
 - [20] C. Armendariz-Picon, V. Mukhanov and P.J. Steinhardt, *Phys.Rev.D***63** 103510(2001).
 - [21] T. Chiba, *Phys.Rev.D***66** 063514 (2002).